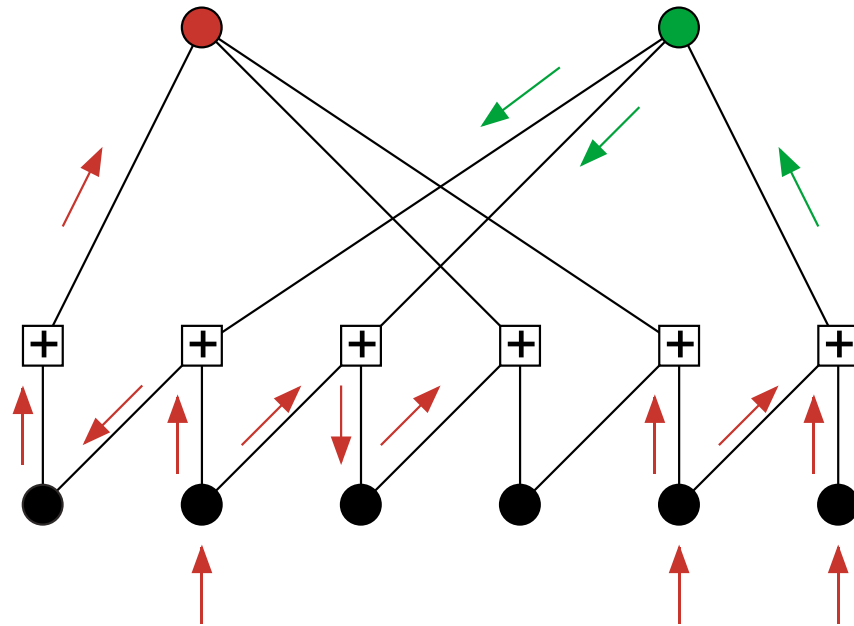


Reliable Communication for the Internet

Hui Jin, Aamod Khandekar, Robert McEliece

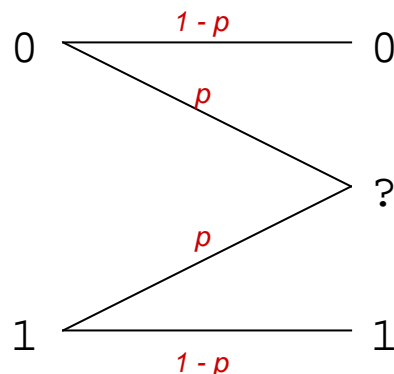
California Institute of Technology
Pasadena, California



Lee Center for Advanced Networking

Pasadena, California, May 16, 2001

The Binary Erasure Channel: A Model for Internet Traffic



- Packets are “erased” with probability p but never received in error.
- The capacity of the BEC is $C = 1 - p$, which is easy to achieve with feedback.
- Without feedback, the classical complexity of communicating at a fraction $1 - \epsilon$ of capacity is $O(1/\epsilon^4)$, obtained by solving linear equations for the erased positions. ■

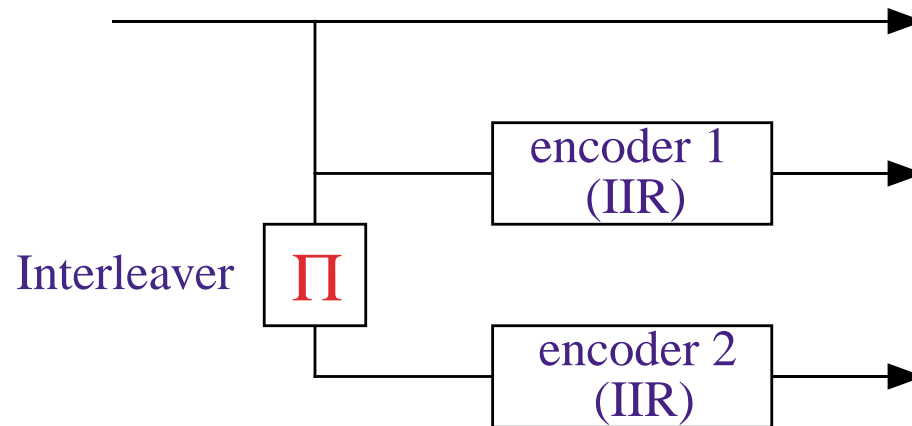
We can Do Much Better with Iterative Message-passing (Turbo) Decoding



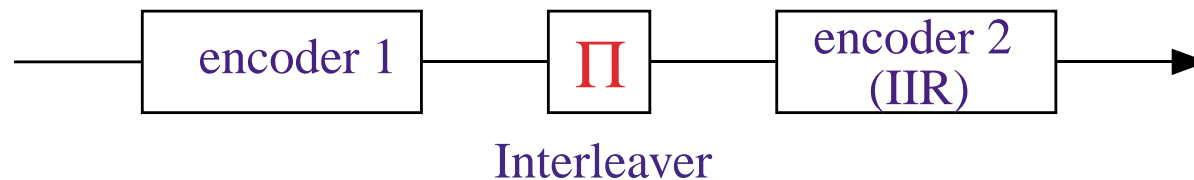
A Low-Complexity Approximation
To “Exact” Decoding.

Codes that can be Decoded in the “Turbo-Style”

- Classical turbo codes:

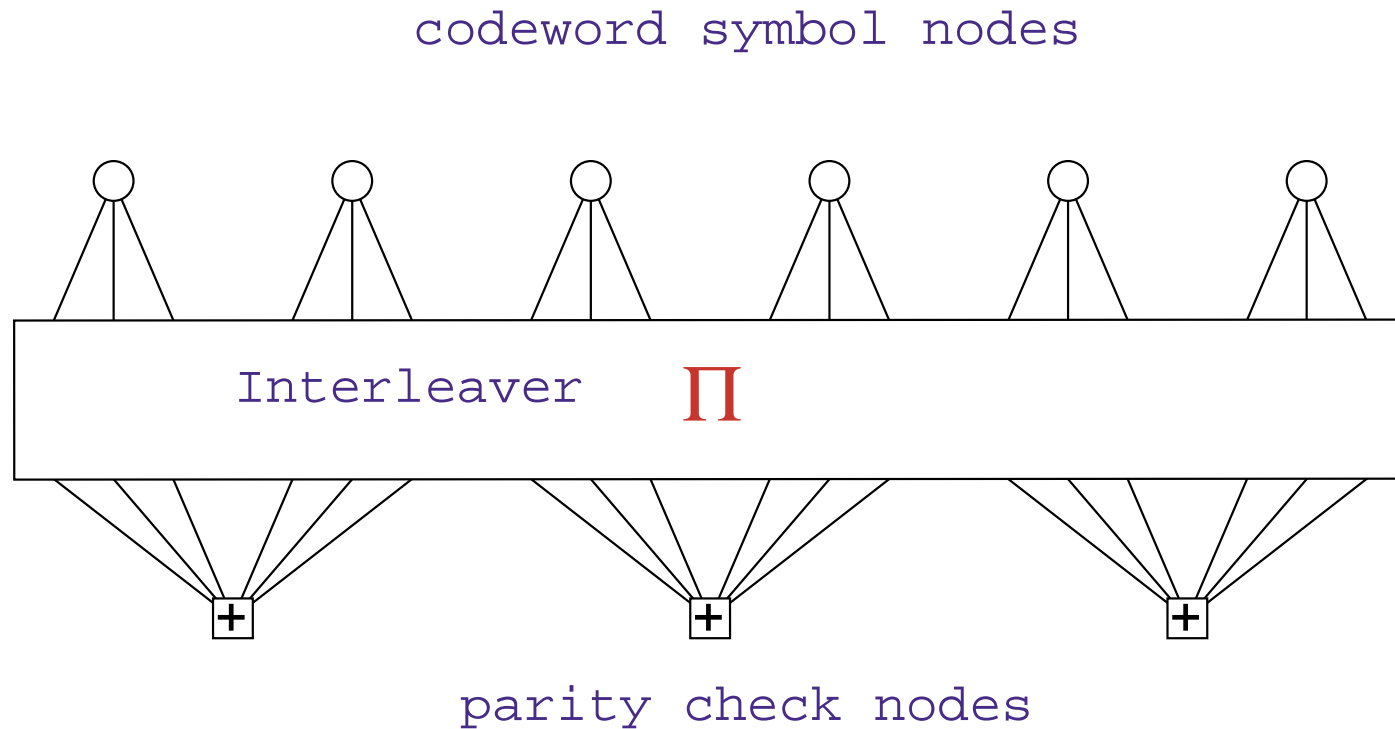


- “Serial” turbo codes:



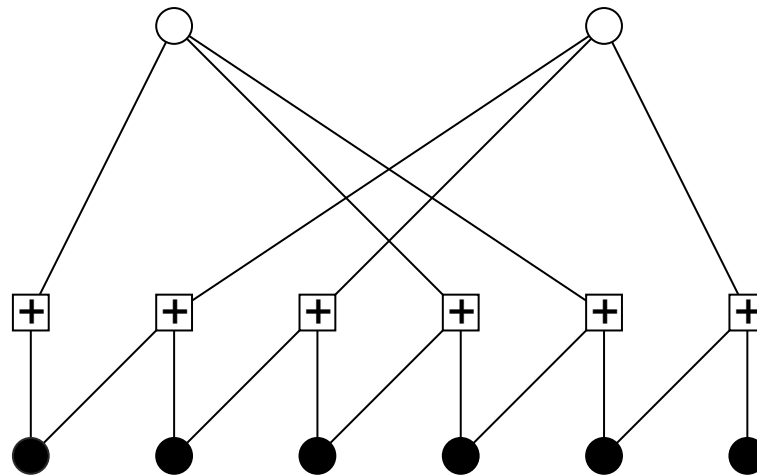
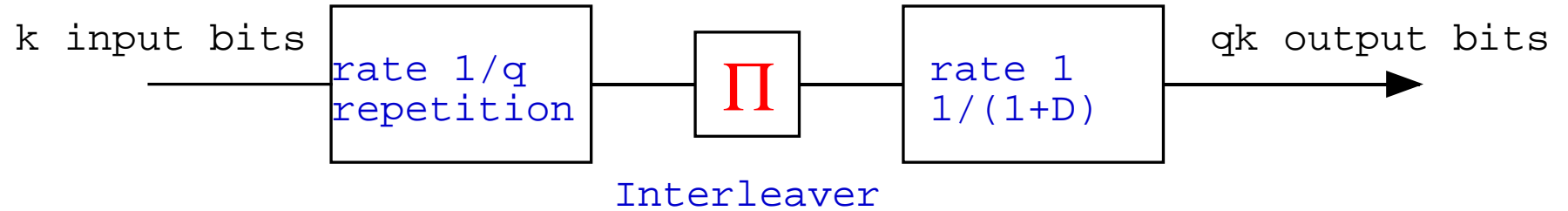
Codes that can be Decoded in the “Turbo-Style”

- Gallager codes (Low-Density Parity-Check), regular and irregular:



And ...

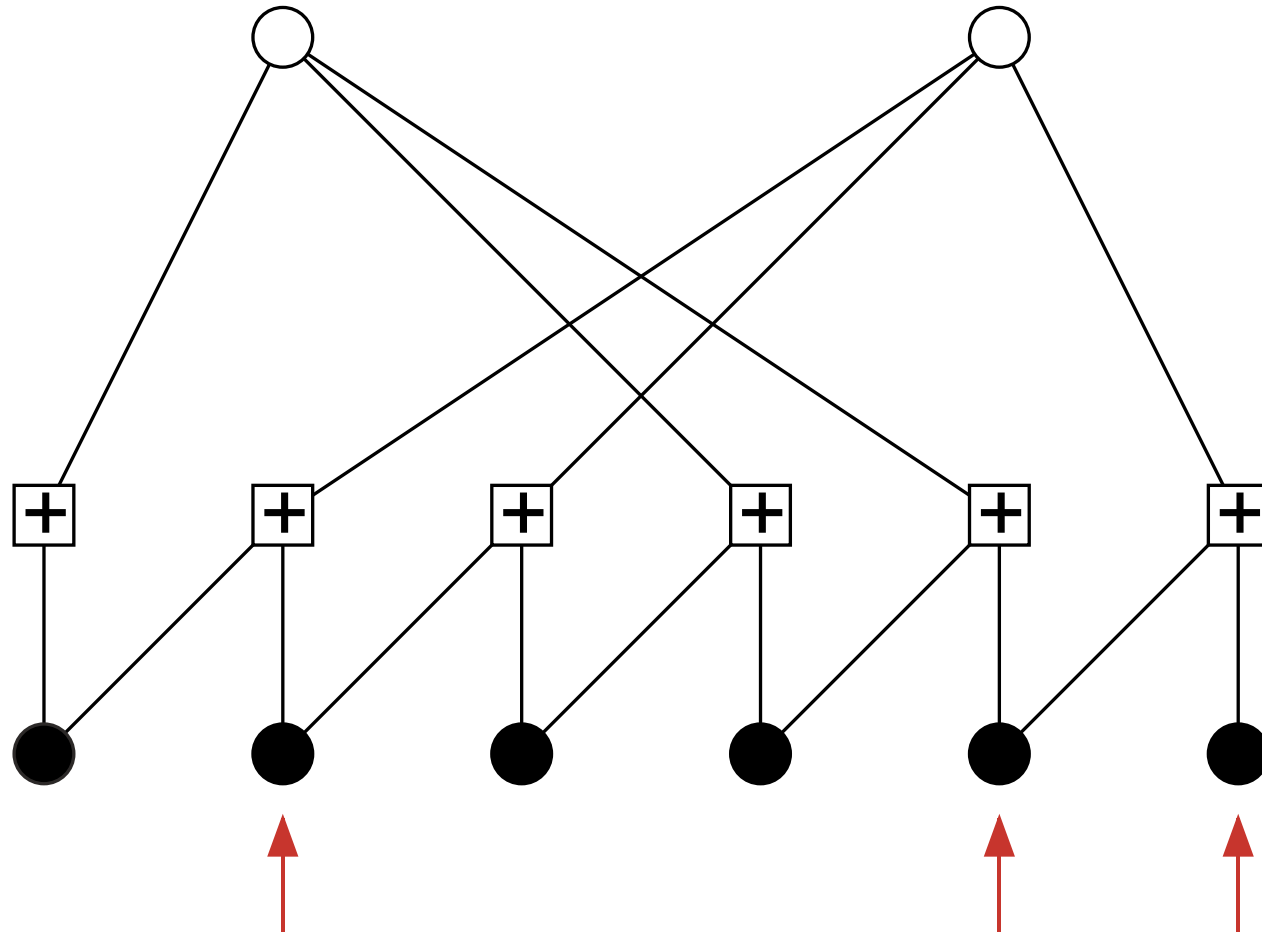
Repeat-Accumulate (RA) Codes



Tanner Graph
($k = 2, q = 3$)

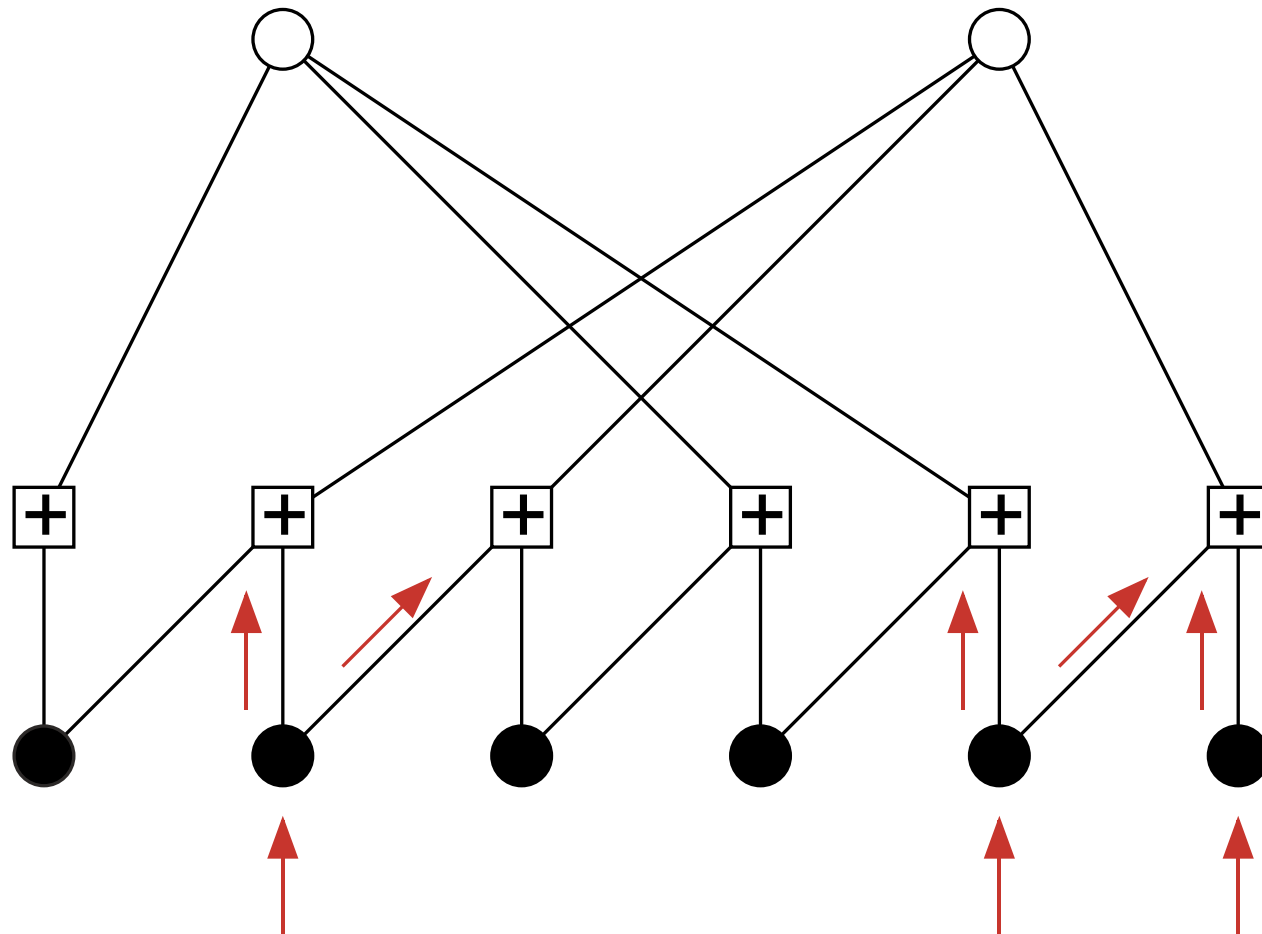
Decoding an RA Code on the BEC Using Message Passing

$\longrightarrow = 1$, $\longrightarrow = 0$



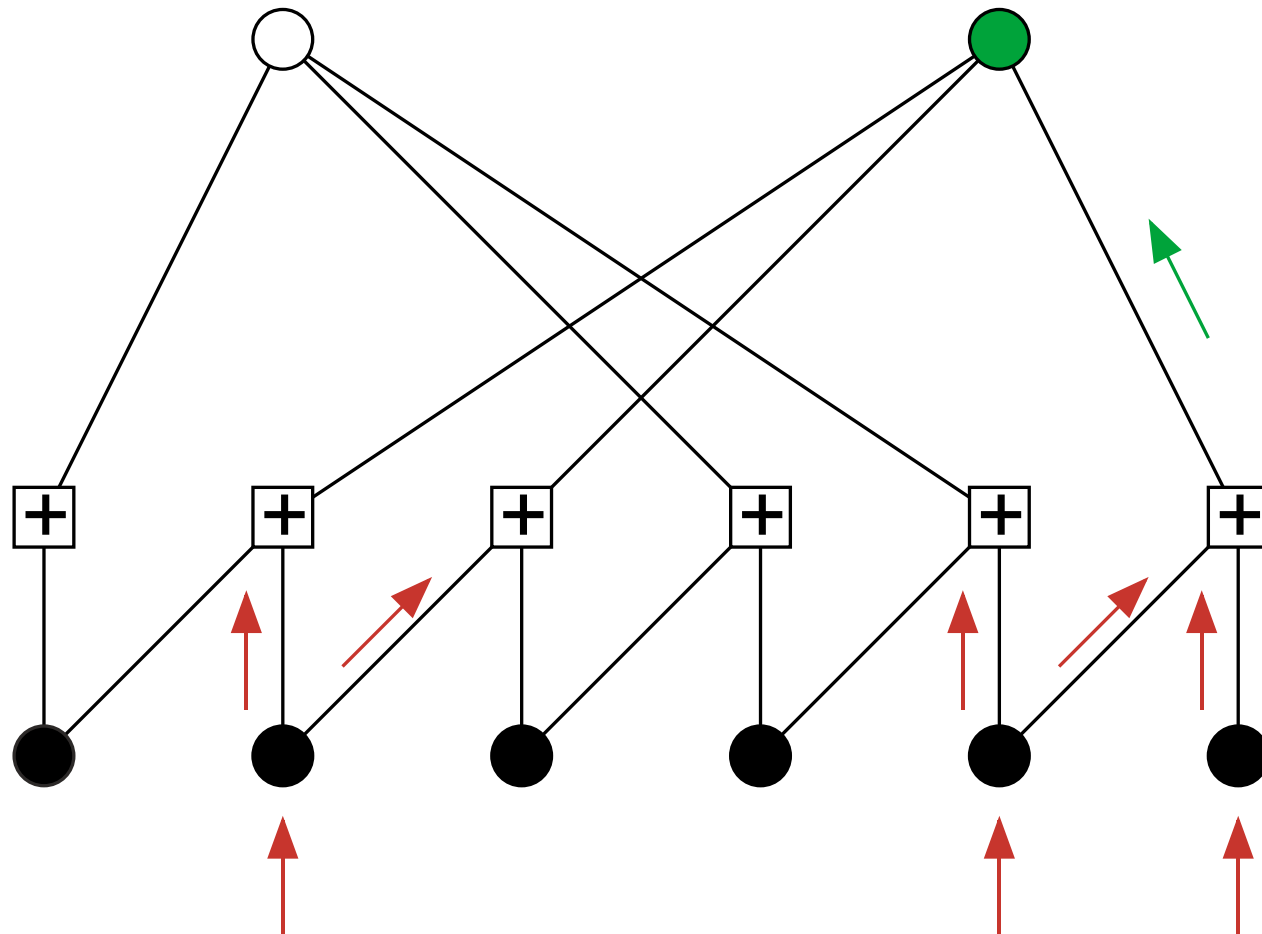
Decoding an RA Code on the BEC Using Message Passing

$\xrightarrow{\text{red}} = 1, \xrightarrow{\text{green}} = 0$



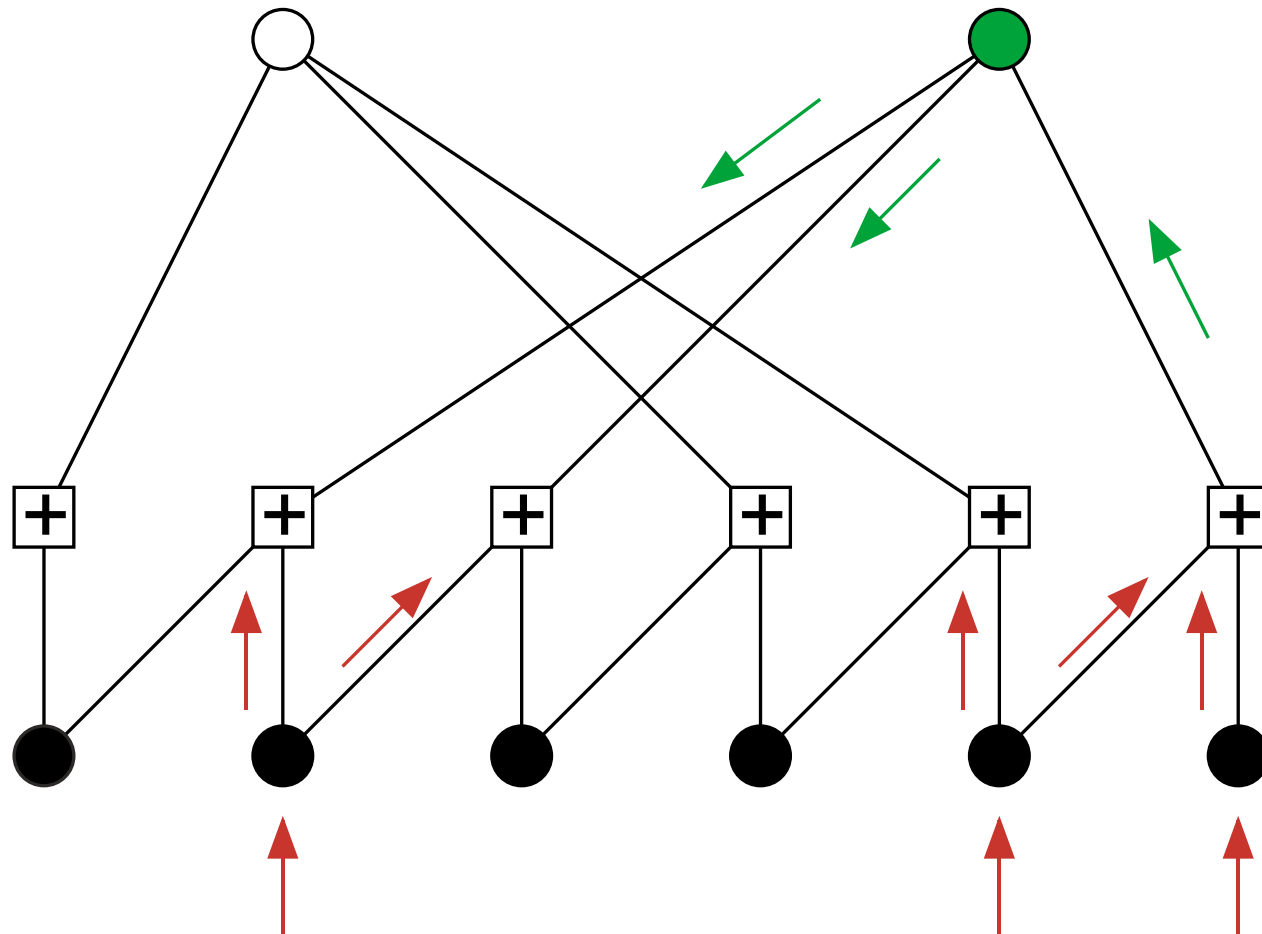
Decoding an RA Code on the BEC Using Message Passing

$\longrightarrow = 1$, $\longrightarrow = 0$



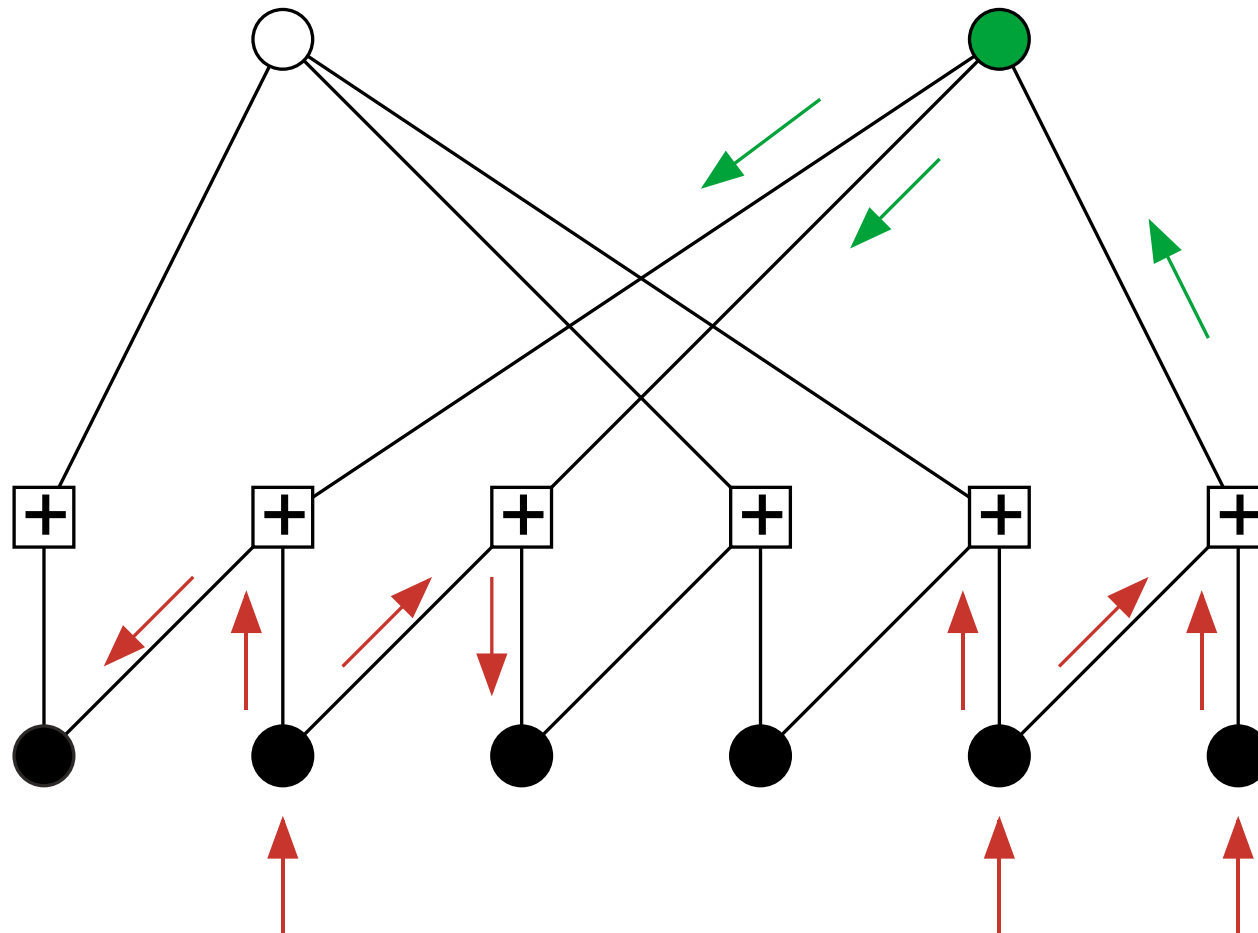
Decoding an RA Code on the BEC Using Message Passing

$\longrightarrow = 1$, $\longrightarrow = 0$



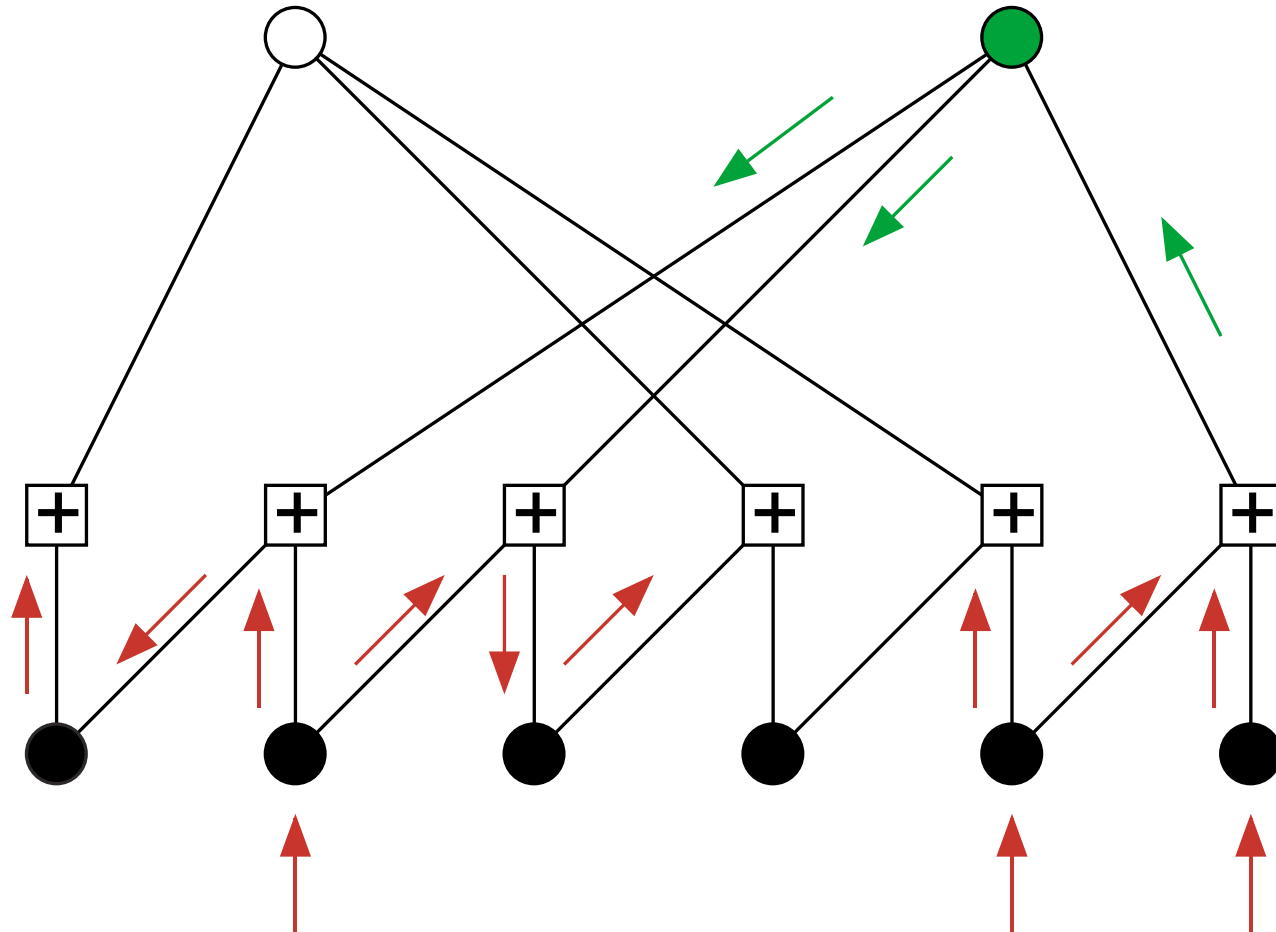
Decoding an RA Code on the BEC Using Message Passing

$\longrightarrow = 1$, $\longrightarrow = 0$



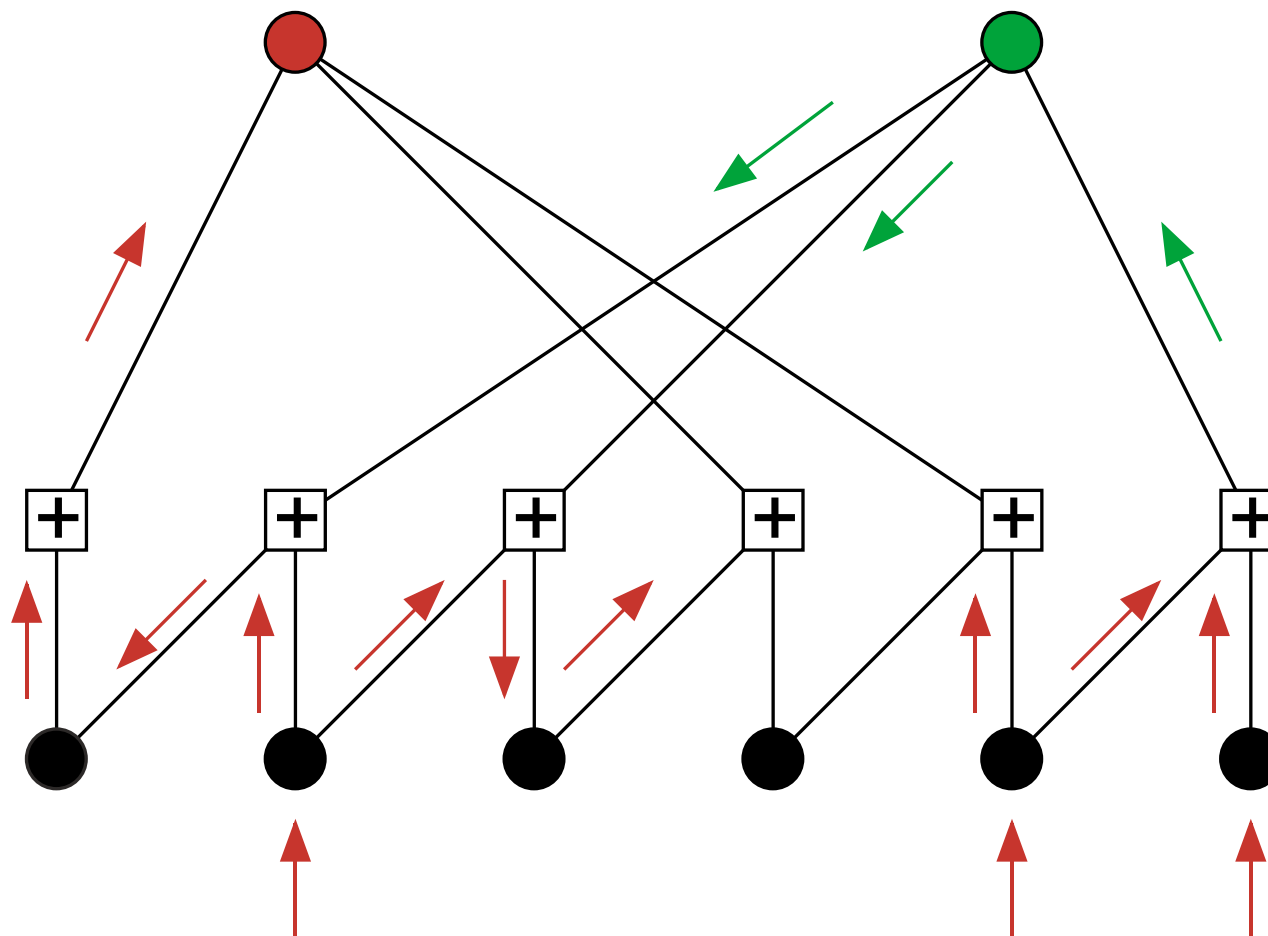
Decoding an RA Code on the BEC Using Message Passing

$\xrightarrow{\text{red}} = 1, \xrightarrow{\text{green}} = 0$

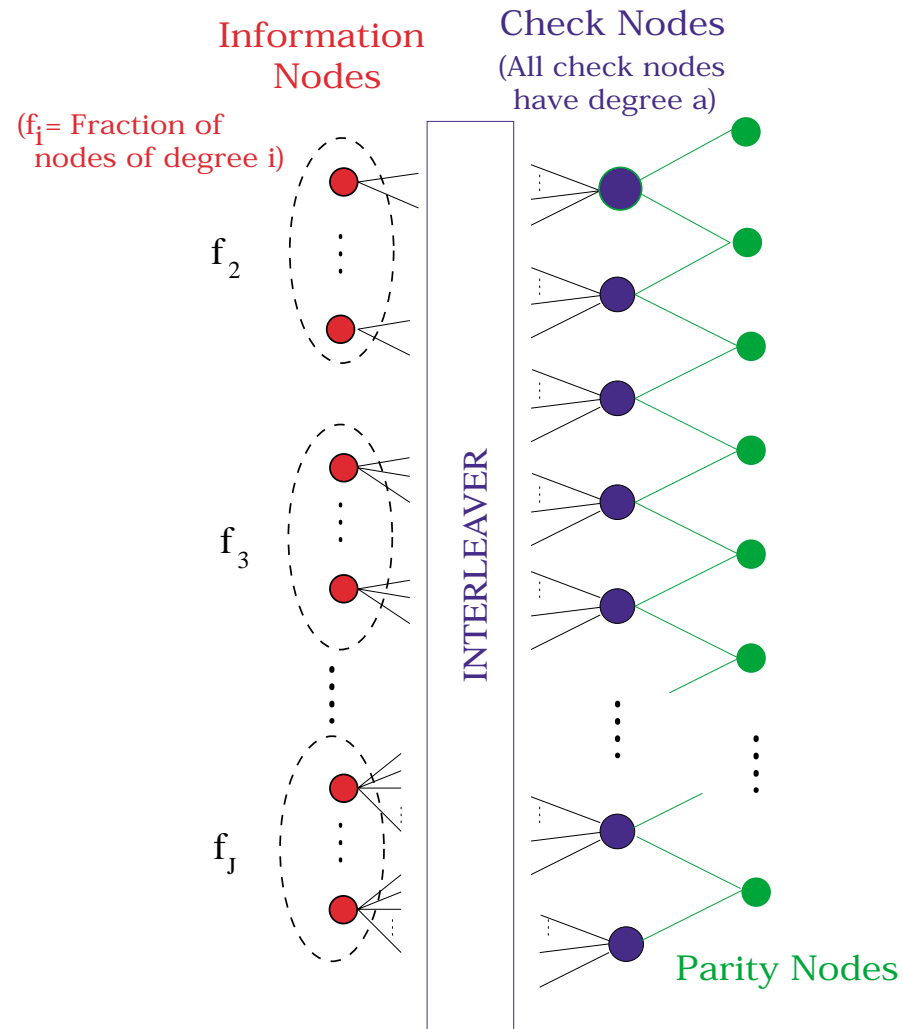


Decoding an RA Code on the BEC Using Message Passing

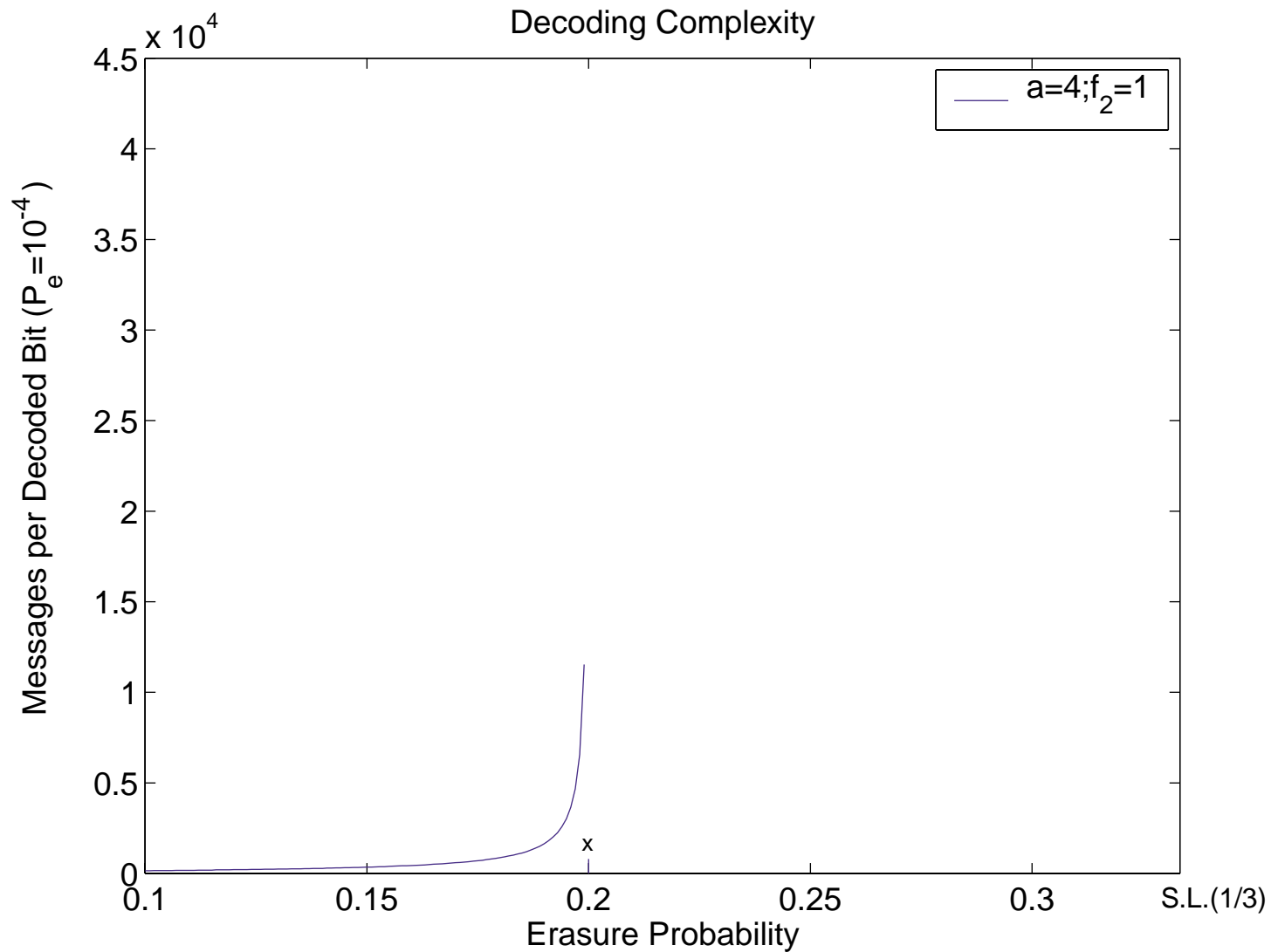
$\xrightarrow{\text{red}} = 1, \xrightarrow{\text{green}} = 0$



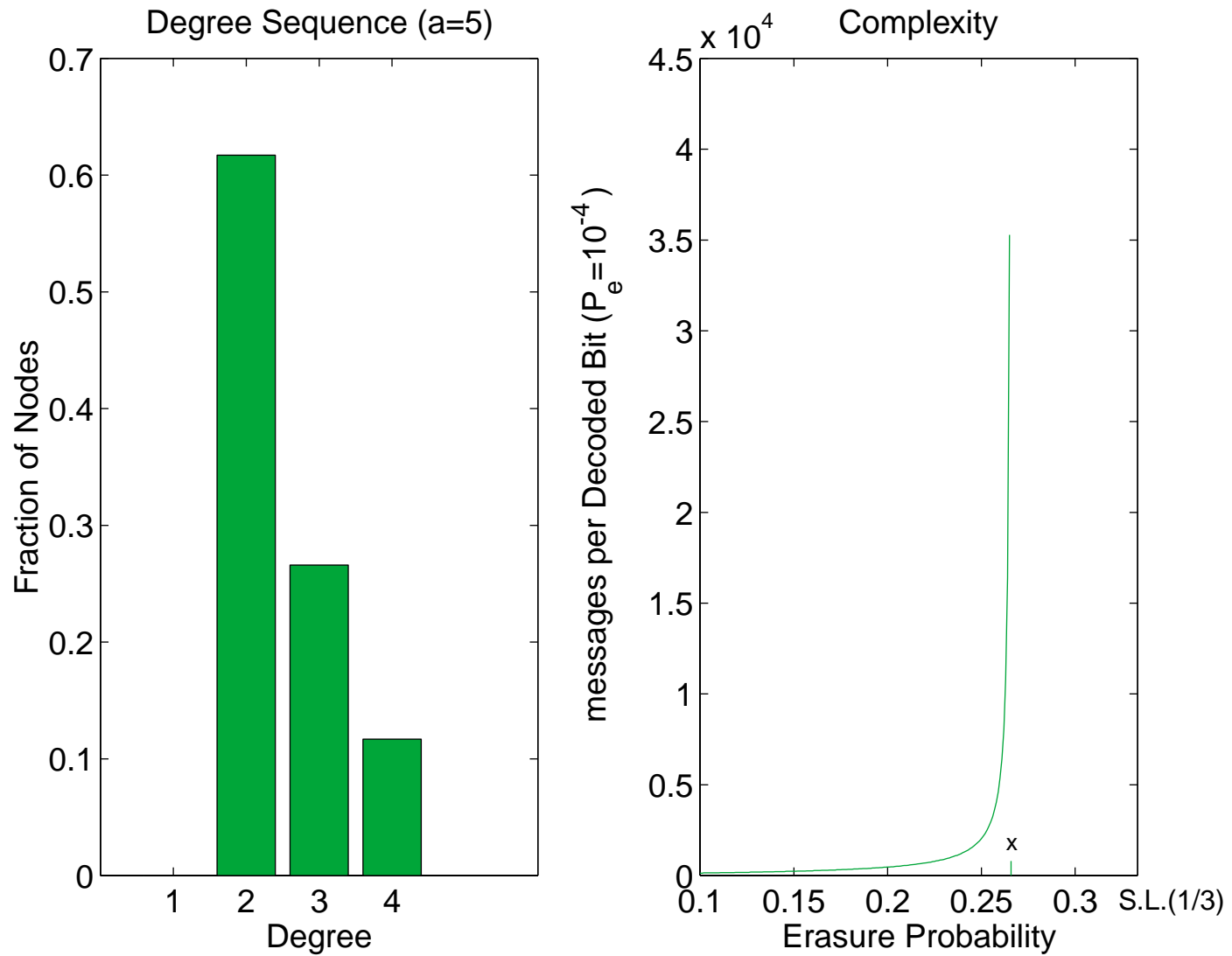
We Can Do Still Better with “Irregular” Repeat-Accumulate Codes



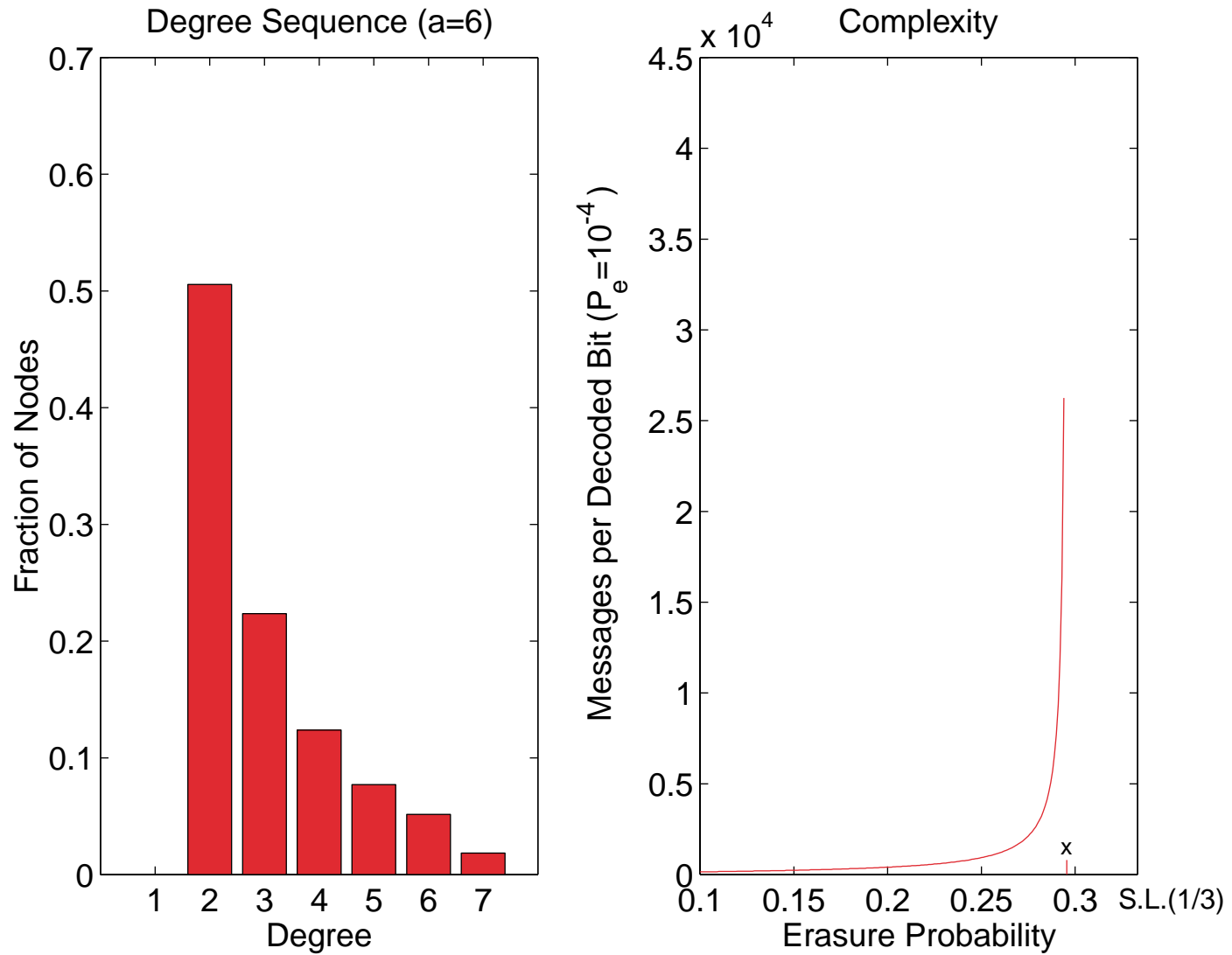
$R = 2/3$ IRA Codes for the Binary Erasure Channel



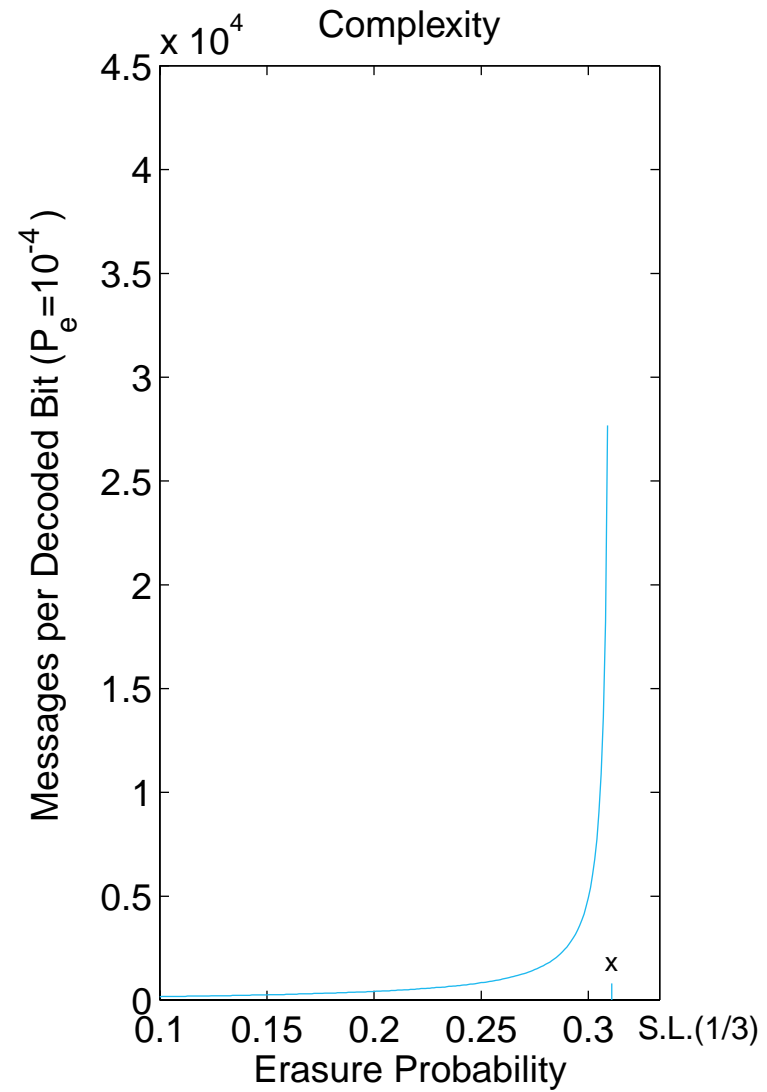
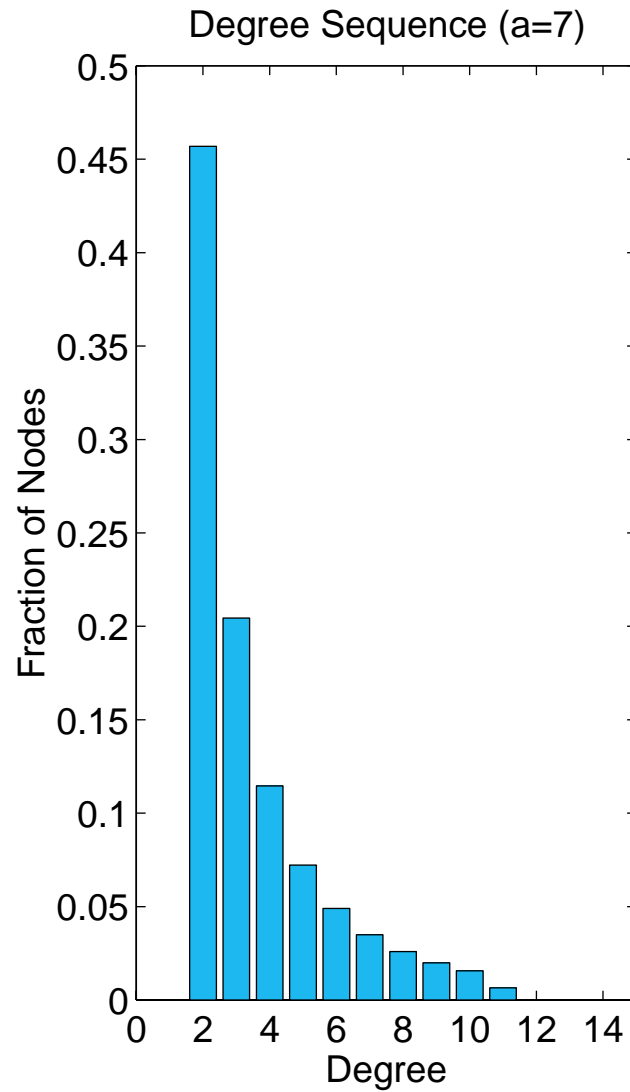
$R = 2/3$ IRA Codes for the Binary Erasure Channel



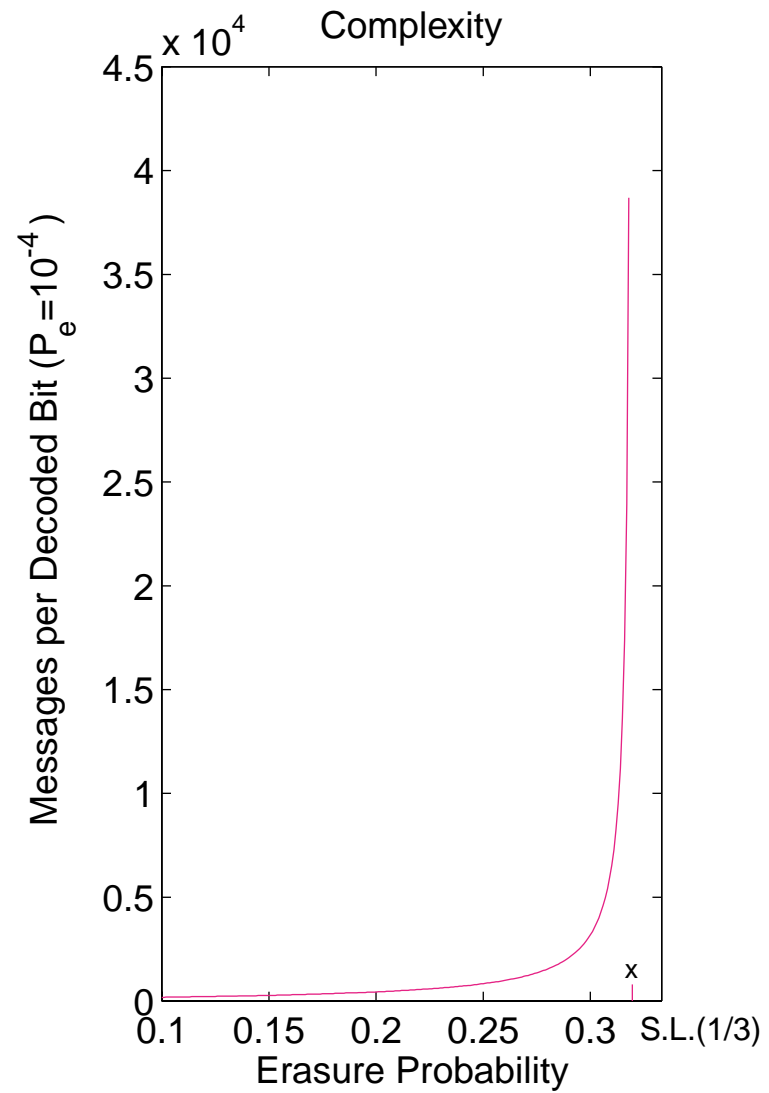
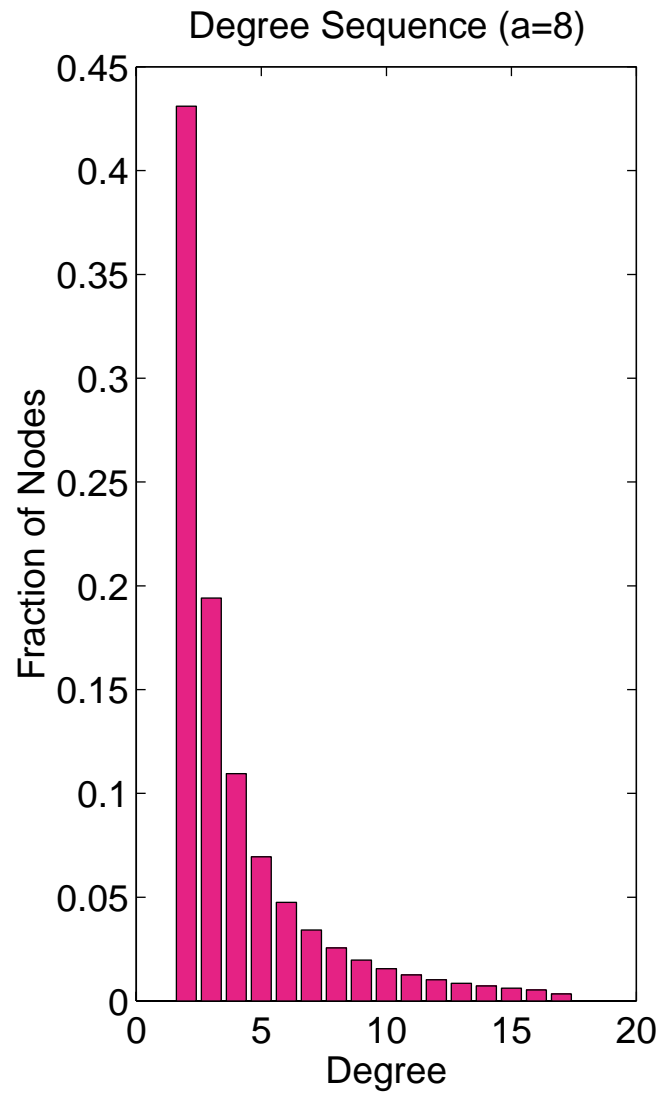
$R = 2/3$ IRA Codes for the Binary Erasure Channel



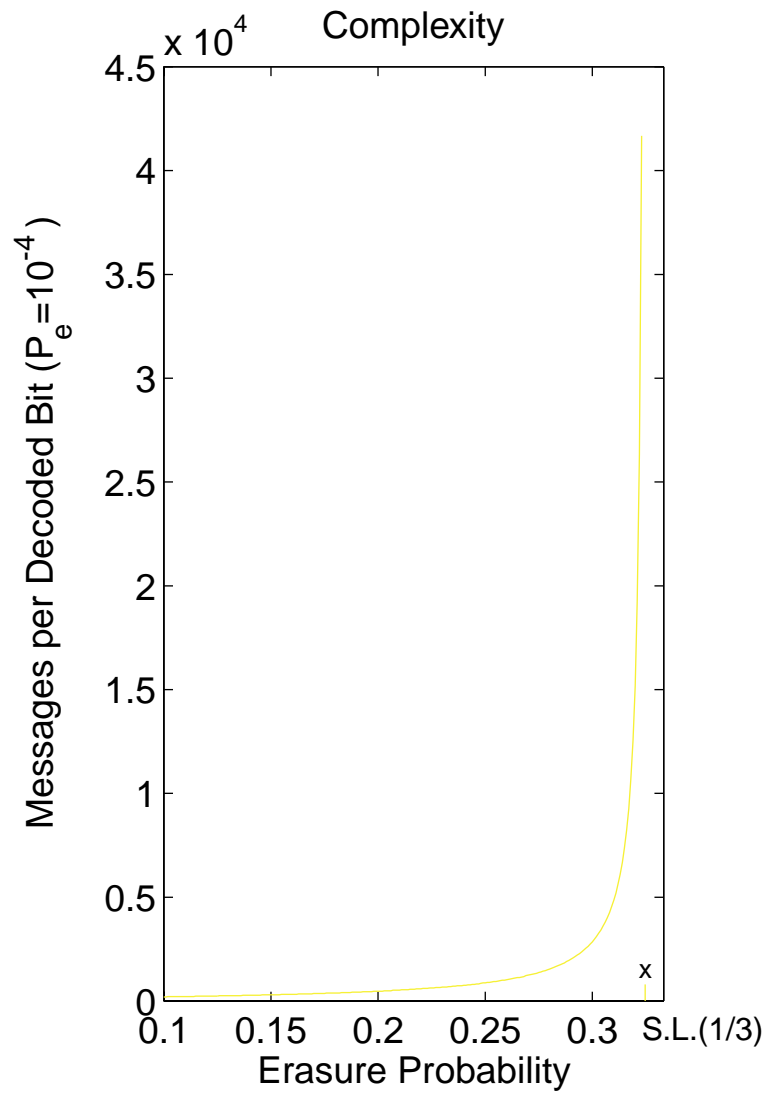
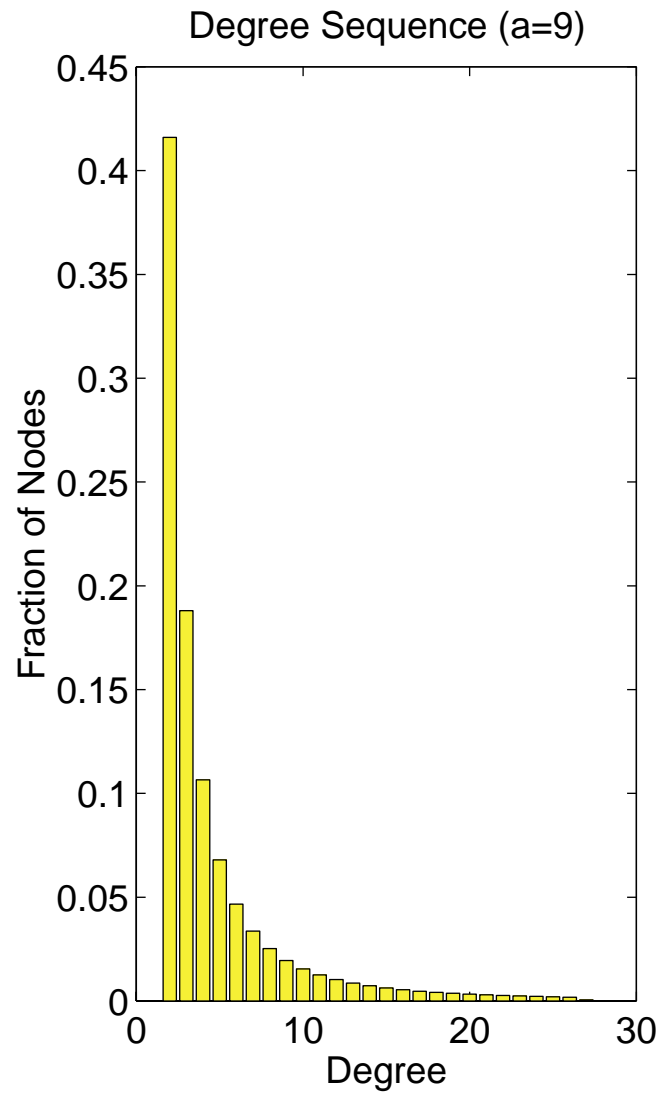
$R = 2/3$ IRA Codes for the Binary Erasure Channel



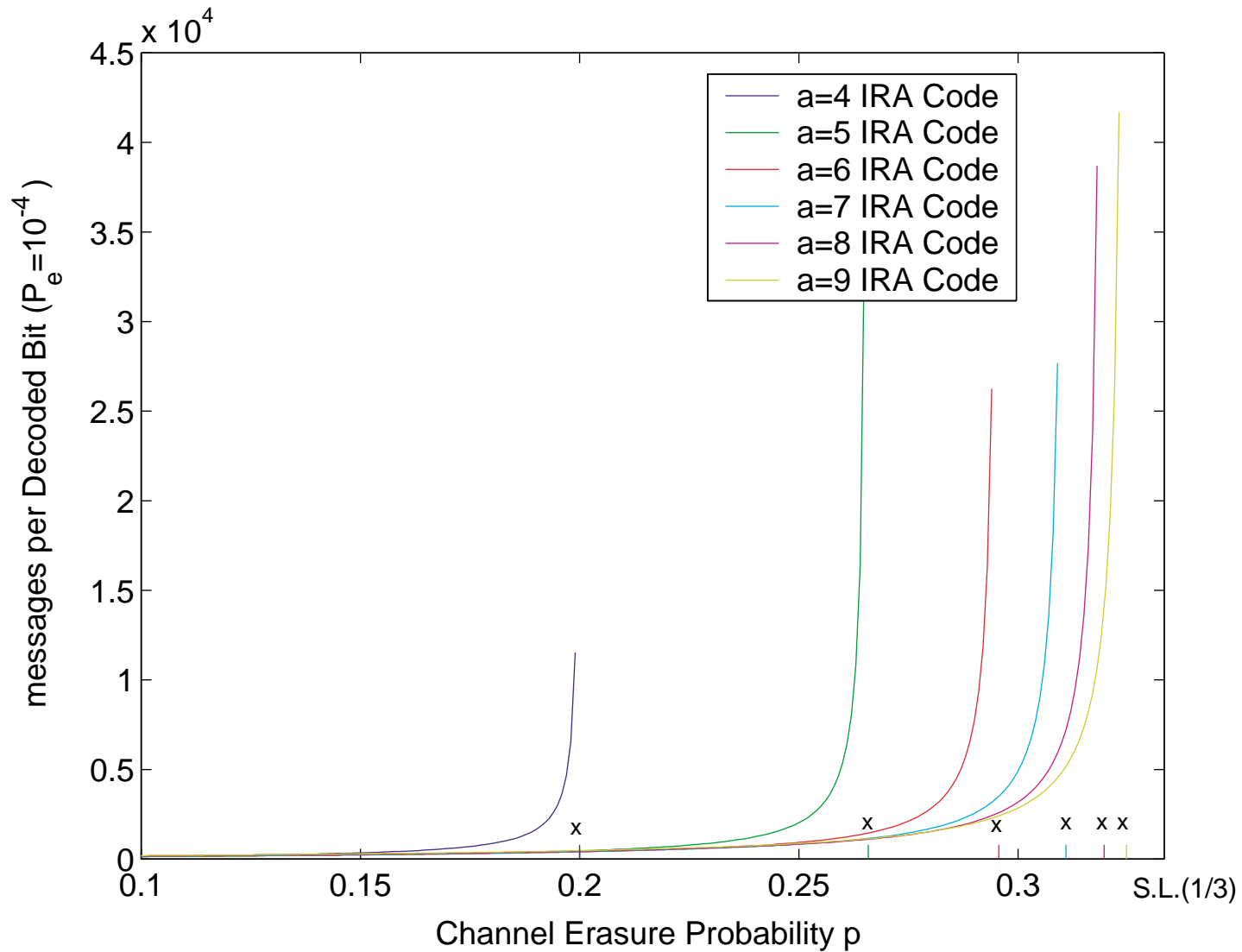
$R = 2/3$ IRA Codes for the Binary Erasure Channel



$R = 2/3$ IRA Codes for the Binary Erasure Channel



$R = 2/3$ IRA Codes for the Binary Erasure Channel



A New Result

Theorem C. *For the binary erasure channel, for IRA codes, the complexity of communicating at a rate equal to $1 - \epsilon$ of capacity is*

$$\bar{\chi}_D(\epsilon) = O\left(\frac{1}{\epsilon^2} \log \frac{1}{\epsilon}\right)$$

Conjecture D. *For the binary erasure channel, for IRA codes, in fact*

$$\bar{\chi}_D(\epsilon) = O\left(\frac{1}{\epsilon} \log \frac{1}{\epsilon}\right)$$

Note: We can prove that $\bar{\chi}_D(\epsilon) = O\left(\frac{1}{\epsilon} \log \frac{1}{\epsilon}\right)$ for irregular LDPC codes.